R300 – Advanced Econometric Methods

PROBLEM SET 1 - QUESTIONS

Due by Mon. October 12

1. Let y and x be scalar random variables. Show that the following hold.

   (i) $\text{cov}(E(y|x), y - E(y|x)) = 0$.

   (ii) $\text{var}(y - E(y|x)) \leq \text{var}(y)$.

   (iii) $\text{var}(y - E(y|x)) = \text{var}(y)$ when y and x are independent.

2. Show that the following functions are probability mass/density functions and compute their first two moments.

   (i) $f(x) = ax^{a-1}$, $x \in (0, 1)$ and $a > 0$.

   (ii) $f(x) = n^{-1}$, $x = 1, 2, \ldots, n$ for n a positive integer.

   (iii) $f(x) = (3/2)(x - 1)^2$, $x \in (0, 2)$.

3. Suppose that, for a scalar random variable x,

$$F(v) = P(x \leq v)$$

is continuous and strictly increasing in v. Define the random variable $y = F(x)$. Derive the distribution of y.

4. For each of the cases below, show that the score is mean zero and derive the efficiency bound for θ.

(i) Binomial with parameters (n, θ).

(ii) Exponential with mean $1/\theta$.

(iii) Exponential with mean θ.